PHASE PLANE ANALYSIS AND OBSERVED FROZEN ORBIT FOR THE TOPEX/POSEIDON MISSION

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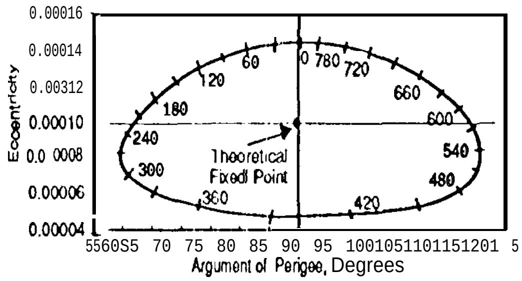
In a frozen orbit the argument of perigee and eccentricit y remain fixed due to the balancing of the secular perturbations of the even zonal harmonics with the long period perturbations of the odd zonal harmonics constant [Chobotov, 1991, chapter 11]. Deviations from this ideal steady state lead to closed curves in the (e,ω) phase plane. These curves can remain nearly closed even under the influence of perturbing forces such as drag and solar radiation pressure. If necessary orbital maneuvers can be applied to recover any drifts duc to these forces. For most frozen orbits, d ıe perigee is frozen at 90", and the eccentricity is very low. In addition, there is a small range of inclinations where frozen orbits have been demonstrated numerically at ω=270° [Smith, 1986] and for highly eccentric orbits (e.g., Molniya orbits). Utilization of the frozen orbit effectively reduces altitude variation over the northern hemisphere as the orbital shape more closely matches the equatorial bulge. The low-eccentricit y frozen orbit was first described for use on SEASAT [Cutting, Born, & Frautnick, 1978] but has also been used or proposed for numerous other missions, including the Atmospheric Explorer (AE) and the Heat Capacity Mapping Mission (HCMM) [Herder, Cullen, & Glass, 1979]; LANDSAT [McIntosh & Hassett, 1982]; GEOSAT [Born, 1987; Shapiro & Pine, 1988]; NROSS [McClain, 1987]; and '1'OPEX/Poseidon [Smith, 1986; Vincent, 1990, 1991; Frauenholz, 1995].

This paper will focus on analytical and numerical treatments of the TOPEX/Poseidon satellite orbit and will usc extensive observations from the primary three-year mission to demonstrate the stability of the frozen orbit and the validity of the analytical treatments. TOPEX/Poseidon was launched by an Ariane 42P on August 10, 1992 with injection occurring at 23:27:05 UTC, approximately 19 min. 57 scc after lift off. The operational orbit was acquired on September 21, 1992, some 42 days after launch, following a sequence of six orbital acquisition maneuvers [Bhat 1993]. The joint US/French mission** is designed to study global ocean circulation



Predicted frozen orbit including perturbations to J20. The tick marks are in days following operational orbii acquisition.

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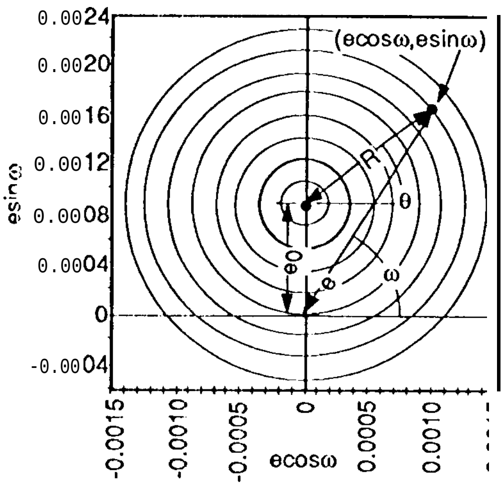
and it.. interaction with the atmosphere to better understand the Earth's climate. This goal is accomplished utilizing a combination of satellite altimetry data and precision orbit determination to precisely determine ocean surface topography. To facilitate this process the satellite is maintained in a nearly circular, frozen orbit ($e \approx 0.000095$ and $\omega \approx 90°$) at an altitude of =1336 km and an inclination of $i \approx 66.04°$. This provides an exact repeat ground track every 127 revolutions (≈9.9 days) and overflies two altimeter verification sites: a NASA site off the coast of Point Conception, California (latitude 34.46910 N, longitude 120.680810 W), and a CNES site near the islands of 1 ampione and Lampedusa in the Mediterranean Sca (latitude 35.54649" N, longitude 12.32054°E).

Previous analytic treatments of the frozen orbit have been performed using J2 and J3 perturbations with numerical extensions to 17th order zonal fields. In the present analysis, stable low-eccentricity frozen orbit solutions will bc analytically demonstrated using a complete zonal expansion of the geopotential field. A general formula for the frozen orbit in terms of the mean elements will bc derived. This analytic solution matches the earlier results with the appropriate truncation. A geometrical interpretation of the frozen orbit will bc provided in terms of trajectories in the phase plane of the non-singular elements $(e\cos\omega, e\sin\omega)$, and these trajectories will bc seen to be nearly circular. This graphical technique will demonstrate that for any specific low eccentricity orbit, there is only onc frozen orbit point, either at the orbital north or south pole, and that the transition between tbc two possible fixed points occurs continuously in the non-singular phase plane as the fixed point crosses the origin. Furthermore, as eccentricity increases, the apparent breakdown of the frozen orbit as ω circulates through the entire range of (O, 360°) occurs when the closed trajectories in phase space enclose the origin. In other words, the concept of a critical eccentricity beyond which the orbit is no longer frozen is a fiction resulting from analyzing the trajectories in an inappropriate phase plane, and the trajectories will always remain closed in the $(e\cos\omega, e\sin\omega)$ phase plane.

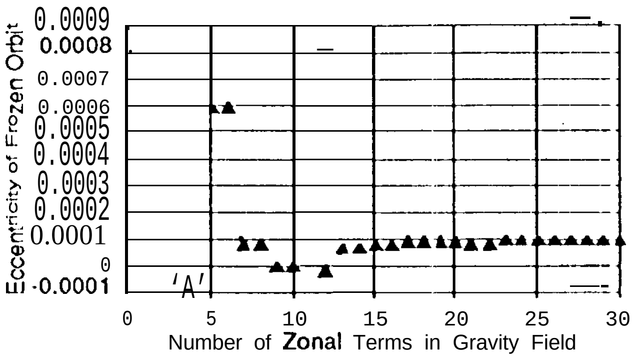


Predicted contours in the nonsingular phase plane to J3. Solutions at ω=270° occur when the center of tho circle falls in the lower half piano.

The general zonal perturbations on the mean eccentricity and argument of perigee arc [Groves, 1960; Merson, 1966]:

$$\frac{d\omega}{dt} = -\sum_{\ell=2}^{\infty} nJ_\ell \left(\frac{R_e\xi}{a}\right)^\ell \sum_{k=0}^{\ell} \cos k\tilde{\omega} \frac{(\ell-1)!}{(\ell+k-1)!} \left\{ \left[\left(\ell + \frac{k}{e^2}\right) P_{\ell-1}^{k}(\xi) + \frac{P_{\ell-1}^{k+1}(\xi)}{e} \right] V_{\ell k}^{0}(i) - 2\cot i\, P_{\ell-1}^{k}(\xi) E_{\ell k 0}^{0}(i) \right\}$$

$$\frac{de}{dt} = -\sum_{\ell=2}^{\infty} \frac{nJ_\ell}{e} \left(\frac{R_e\xi}{a}\right)^\ell \sum_{k=1}^{\ell} k u_k \sin k\tilde{\omega} \frac{(\ell-k)!(\ell-1)!}{(\ell+k)!(\ell+k-1)!} T_\ell^k(0) T_\ell^k(\cos i) P_{\ell-1}^{k}(\xi)$$

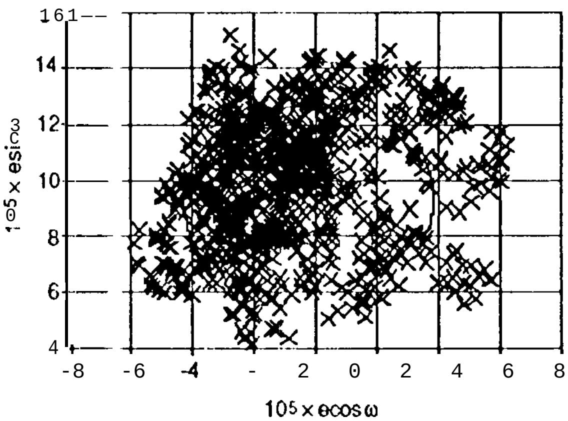


Critical point eccentricty as a function of number of zonal harmonics in gravity expansion. Positive values indicate ω=90°, while negative values indicate ω=270°.

where the $T_\ell^k(x) = (1-x)^{k/2}\frac{d^k}{dx^k}P_\ell(x)$ and the $P_\ell^k(x) = (-1)^{k/2}T_\ell^k(x)$ are associated Legendre polynomials, and the variables $\xi = \left(1-e^2\right)^{-1/2}$ and $\tilde{\omega} = \pi/2-\omega$. This gives two nonlinear differential equations in two unknowns, e and ω.* The system can be linearized about the steady state; the eigenvalues of the Jacobean of the linearized system, evaluated at the fixed point.., determine the stability of the nonlinear system in some neighborhood about the steady state. This procedure is used because it is usually not possible to solve the nonlinear system explicitly. This complete stability analysis will be given. The standard method of analysis is as follows. First, determine the location of any fixed points, or steady states, (the solutions of e = $\dot{\omega}$ = O) of the system. This occurs when $\cos\omega$ = O, i.e., ω=90° or ω=270°. Determination of the corresponding eccentricity is more complicated, and will be derived in detail in the paper. The result is

$$e_{ss} = \frac{-\sin i \sum_{\ell\ odd} J_\ell\left(\frac{R_e}{a}\right)^\ell\left(\frac{\ell-1}{\ell+1}\right)P_{\ell-1}(0)P_\ell'(\cos i)}{\sum_{\ell\ even} J_\ell\left(\frac{R_e}{a}\right)^\ell P_\ell(0)\left[\frac{\ell(1+\ell)}{2}P_\ell(\cos i)+\cos i\,P_\ell'(\cos i)\right]}$$

This solution is valid except for a regime very close to cosi $\approx 1/\sqrt{5}$ ($i\approx63.4°$), where the small eccentricity approximation fails. This so-called *critical inclination* will be explored in the paper. When $e_{ss}<0$, the frozen orbit occurs at ω=270° and e $= -e_{ss}$e= -ess. These solutions correspond to the center of the contours falling on the negative y axis in the $(e\cos\omega, e\sin\omega)$ phase plane. The J3 approximation reduces to e. $= -J_3R_e \sin i/2J_2a$ and has a period of $2\pi\Big/\left\{\frac{3nJ_2R_e^2}{a^2}\left(1-\frac{5}{4}\sin^2 i\right)\right\}$ as has been previously demonstrated. The J3 contours in the nonsingular phase plane are described by the equation $X^2 + (y-e_o)^2 = R2$ and in traditional



Observed eccentricy and argument of perigee for the TOPEX/Poseidon orbit.

* The mean inclination is slowly varying with respect to e and $\omega\left(di/dt = -e\cot i(de/dt)/\xi^2\right)$, and $da/dt = O$. Other functions used are defined as uk $=2-\delta_{k0}$, $V_{\ell k}^0(i) = u_k\frac{(\ell-k)!}{(\ell+k)!}T_\ell^k(\cos i)T_\ell^k(0)$ and $E_{\ell k0}^0(i) = \frac{1}{2}u_k\frac{(\ell-k)!}{(\ell+k)!}T_\ell^k(0)\left[k\cot i\ T_\ell^k(\cos i)-T_\ell^{k+1}(\cos i)\right]$

coordinates $e^2 - 2ee_o \sin\omega + e_0^2 = R^2$.

This paper is relevant to the astrodynamics, guidance and control, and remote sensing technical sessions of the conference. It extends the proof of the existence of low-eccentricity frozen orbit to a complete zonal geopotential and derives and explicit formula for the frozen eccentricity as a function of the gravity field. The low eccentricity frozen orbit is extremely useful for remote sensing satellites such as those in the Mission to Planet Earth, as the altitude variation and hence variability in observation conditions is minimized. Furthermore, the paper will be supplemented with extensive observations from the TOPEX/Poseidon mission. These observations will be compared with the analytic and numerical predictions, and will demonstrate the possibility of maintaining an extremely low eccentricity orbit for several years.

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